Math's Turn

Changing a History of Math in Games
to a Future of Games in Math

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Abstract

Math is so intertwined in games that we take it for granted. Likewise, teachers often take for granted how instrumental games can be in their classroom instruction. When they do use games, often it is mainly to take up time or for pure fun (without much, if any, educational purpose). This paper makes the argument for more purposeful games and their regular use in the classroom. With so many students falling further and further behind in math over the course of their school-age years (particularly in these pandemic times), it is imperative that we use more effective tools to get them on track. Games can be one of those tools.

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Introduction

Starting as early as preschool, we are introduced to a wide variety of games in the course of our educational careers. Many of these games serve the purpose of entertainment, but in many circumstances they are also used for learning. One common element that you will find across the board in these games is their surprisingly close relationship with math and its concepts.

With the thousands upon thousands of games that have been introduced over time, it is difficult to think of any that do not involve math to some degree. In fact, some games are completely based on math, even though we may not fully see the concepts at work in the background. For instance, blackjack is a game full of addition and probability. From the time that the first card is dealt until the game ends, a winning strategy depends on odds and calculated advantages.

With so many games reliant on math to be successful, I would argue that perhaps it is time to explore more fully how games can help students be more successful in math, particularly when they are behind in expected skills for their current math class. When a teacher is faced with students who are behind or with students who just despise the subject, how might this form of enjoyment be turned into a form of learning? I believe it can and should be leveraged to great advantage in mathematics and statistics classrooms. In this 501 curriculum project, I will describe the interrelationship between games, mathematics, and mathematics pedagogy. I will develop and implement several classroom activities that I believe have the potential to engage students in thinking mathematically while interacting with peers in an enjoyable context of recreational games.

A Preliminary Look at the Use of Games in the Classroom

Regardless of their background, I believe that most, if not all, children enjoy playing games. Mathematics teachers should take advantage of this knowledge and utilize instructional games to engage children in the learning process. A well-constructed mathematical game can create a high-cognitive demand task while creating friendly competition that will hold students' attention.

In his journal article on the use of games as a means for teaching mathematics, Ernest (1986) argued that games can be used to teach mathematics effectively by providing reinforcement and practice of skills, by providing motivation, by helping in the acquisition and development of concepts, and by developing problem-solving strategies (p. 3). In addition, teachers can also use games to spark interest in students, to retain attention in the classroom and to produce group interaction.

Though games can achieve all these goals and more, when it comes to mathematics learning, it is important that the objective and rules of the games involve mathematical concepts in order to be effective tools for learning. While playing a round of Jeopardy or bingo using math questions can be fun and useful for the reasons mentioned earlier, games where mathematical analysis is in the blueprint of the game design more naturally allow students to learn and perform mathematical processes. **CandyBot**, an app developed at Virginia Tech that focuses on fraction and function skills, and **Set**, a logical-reasoning card game found on the website setgame.com, are great examples of these high-cognitive demand games. The productive struggle these games provide can be just what teachers need to reach their students. When math classes focus on developing critical thinkers –

even with the more unsuccessful or disinterested students – tools that promote creative problem-solving skills provide a path to success.

It is my hope that game time in math classes goes beyond just using mathematical content and processes. I would like to see more math-based games constructed and used in order to bring more value to instruction, remediation and skill-building. According to Kapp in *The Gamification of Learning and Instruction* (Kapp, 2012, p. 7), desirable attributes of a mathematical game include:

- a challenge against one or more opponents (but no more than four so that the next turn comes quickly and students stay engaged)
- a clear set of rules based on mathematical reasoning
- a set of goals throughout the game culminating with a larger goal at the end
- a reasonable length of time to start and finish the game during class time (but should not just be a time-filler)
- a decision exhibiting mathematical skills be made to move ahead
- a learned objective with each decision/move
- a finite number of moves to reach the goal
- an end to the game based on the goal being reached.

Designing games that achieve all these factors requires a high degree of thoughtfulness and creativity but is well worth the reward of student learning and teacher development.

A History of Probability: Borne from Games

Though it is easy to find a history of math in most games, some of math's history has come from games as well. In *Calculus, Volume II* by Apostol, the author traces the history of a mathematical theory of probability back to a gambling dispute in 1654. Two famous French mathematicians, Blaise Pascal and Pierre de Fermat, were playing a game that consisted of throwing a pair of dice 24 times and deciding whether or not to bet money on the occurrence of at least one double-six during the 24 throws. Antoine Gombaud, Chevalier de Méré, a French man with an interest in gaming and gambling questions, brought Pascal's attention to a discovery he made concerning the popular dice game. A rule of thumb used by gamblers of that time (much like the blackjack hit charts of today) initially led de Méré to believe that betting on a double six in 24 throws would be a good idea, but his own calculations showed the opposite. When a die is thrown four times, the probability of getting at least one six is a little more than one half, producing favorable odds. Knowing that result, de Méré thought rolling two dice 24 times would create the same odds. However, throwing a pair of dice 24 times produces a probability of less than one half for getting at least one double six (Figure 1). He had not taken into account the difference in sample spaces when throwing one die (sample space of 6) versus two (sample space of 36) and the need for both sixes to happen at the same time, a factor not included in the one die scenario.

$$
\begin{aligned}
\Pr\{\text{at least one six in 4 throws}\} &= 1-\Pr\{\text{no six in all 4 throws}\} \\
&= 1-(5/6)^4 \\
&= .518.
\end{aligned}
$$

$$
\begin{aligned}
\Pr\{\text{at least one double-six in 24 throws}\} &= 1-\Pr\{\text{no double-six in all 24 throws}\} \\
&= 1-(35/36)^{24} \\
&= .491.
\end{aligned}
$$

Figure 1. Calculated probability of getting at least one six when throwing one die four times, versus the probability of getting a double six when throwing a pair of dice 24 times.

As de Méré continued to lose bets on double six, he started to realize that the probabilities must not be the same. He called on Pascal to help him figure out this discrepancy. In so doing, Pascal assisted with laying the groundwork for modern probability theory.

Prior to this event, some other games of chance had been calculated but no fully developed theory had been established. However, the results of Pascal's work prompted the Dutch scientist Christian Huygens to publish the first book on probability. Problems found in gambling were the central focus of the book, and it created much interest during that time due to the popularity of games involving chance and odds. Interest in probability concepts continued to grow from there. In 1812, Pierre de Laplace moved away from seeing probability as just a tool for gaming and presented a great deal of new information to add to Huygens's work. He approached probability theory through scientific and experimental problems. From there, this branch of mathematics exploded and several subareas emerged such as statistical mechanics and actuarial mathematics. Statistics is a well-known branch of mathematics in and of itself but it was born from probability and now can be found in other disciplines such as psychology and social science.

It is quite interesting to see the journey of probability from a game of chance to influencing other areas of academia that are not directly related to math. Students often engage in learning subjects without giving much thought to how they were created or designed. This research was enlightening to me and I am sure most students of probability are not aware of it. Beyond academia, probability is hugely connected to insurance, weather forecasts and…games. One such game is blackjack.

A History of Blackjack: A Game Borne from Probability

The origin of blackjack is debated, and the rules have changed since its beginnings. It was actually called 21 initially, signifying its deep roots in numbers and math. Some researchers believe the game came to fruition in 1700 at casinos in France. It was called Vingt-et Un which translates to twenty-one. Other researchers say that the Romans created the game as they were known for having a great love of gambling. In Spain, another version of the game called One and Thirty was played where the goal was to reach 31 with at least three cards. Eventually, the game found its way to the United States in the 18th century and has become one of the most popular games on casino floors.

As gambling became legal in the state of Nevada in 1931, 21 started to become more and more popular there. To get more people to play, some casinos would offer special payouts for having certain cards: the Jack of Spades and the Jack of Clubs, better known as the black jacks. Ultimately, the game became a player favorite and no longer needed an extra incentive to get participants, so casinos did away with the special payouts but the name Blackjack stayed around and is still used to this day. Certain aspects of the game have changed as well, such as players being allowed to double down whereas only the dealer could before and all players at the table can play in each round unlike the past when one round was just between one player and the dealer (The World of Playing Cards, 2010). Blackjack continues to evolve in some ways, particularly in these times of online gaming. However, one aspect about it that has not changed over time is its reliance on probability. Some players view the game of blackjack only in terms of the simple math operation of addition, but there is much more to it. Probability is at the foundation of the game and is the reason why casinos make large sums of money from their patrons. In California, it is illegal for casinos to directly profit from patron losses so they have to allow a third-party corporation to handle the wins and losses at each table. They, in turn, pay a large sum to the casino for accepting their bid for the job. The third-party corporation hires a team of "players" to sit at each table and play the games using specific moves for every possible scenario. One of my jobs right out of college was working as a "player" in various casinos in California. The game all of my co-workers and I were required to learn how to play first was blackjack. We were given a chart similar to the one in Figure 2 and required to memorize all the possible hand plays within a couple of weeks. I witnessed co-workers get questioned, written up and even fired for not consistently implementing the chart strategy. This is a well-known practice in the casino industry and emphasizes the importance of probability in the game and to keep that job.

	Dealer Upcard									
Hard Total	2	3	4	5	6	7	8	9	10	A
5-7	H	H	H	H	H	H	H	H	H	H
8	H	H	H	H	H	H	H	H	H	H
9	H	D	D	D	D	H	H	H	H	H
10	D	D	D	D	D	D	D	D	H	H
11	D	D	D	D	D	D	D	D	D	D
12	H	H	S	S	S	H	H	H	H	H
13	S	S	S	S	S	H	H	H	H	H
14	S	S	S	S	S	H	H	H	H	H
15	S	S	S	S	S	H	H	H	R	R
16	S	S	S	S	S	H	H	R	R	R
17	S	S	S	S	S	S	S	S	S	RS
Soft Total	2	3	4	5	6	7	8	9	10	A
A,2	H	H	H	D	D	H	H	H	H	H
A,3	H	H	H	D	D	H	H	H	H	H
A,4	H	H	D	D	D	H	H	H	H	H
A,5	H	H	D	D	D	H	H	H	H	H
A,6	H	D	D	D	D	H	H	H	H	H
A,7	DS	DS	DS	DS	DS	S	S	H	H	H
A,8	S	S	S	S	DS	S	S	S	S	S
A,9	S	S	S	S	S	S	S	S	S	S

Figure 2. Strategy chart used for playing the various hands of blackjack.
Soft 17 is the combination of an ace and 6 hand which can be viewed as 7 or 17.
H= hit, D= double down, R= surrender, S= stand, RS= surrender if possible, otherwise stand, DS= double down if possible, otherwise stand.

Most people would be very surprised to hear so much math and thought is involved in blackjack. With all the money lost in casinos each day, it should come as no shock that they have an intricate strategy for winning. In any fair game involving two parties or options, you expect there to be a 50/50 chance of either person winning or either option occurring. When a game involves circumstances where one of the players or options consistently receives more than 50% of the wins, an advantage has been created. The house advantage in most casinos is about 8% (so 54% total versus the common player's 46%). Depending on the number of decks used, this percentage can go up or down (more decks create more of an advantage for the dealer). Much of the advantage comes from their position as the last player on the table since some players bust (exceed 21) and lose their money before the dealer's turn even comes around. The rest of the advantage comes from strategizing through probability. If a player can learn and implement this same strategy, he or she can reduce the house's advantage to under 1%. Advanced practices can even result in an advantage to the player. Figure 3 shows the player's advantage based on using basic strategy and the dealer's hand.

Player Advantage vs. Dealer Up Card

The first two columns in this odds chart explain the dealer's chance of busting, depending on the up card showing. Note that the dealer has the highest chance of busting when showing a 5. The third column in this chart shows the player advantage of using basic strategy, compared to each up card the dealer is showing (expected value). You can see that the player has the highest advantage of 23.9%, when the dealer is showing a 6. When the dealer is showing any card that is 9 or higher, the player is in the negative advantage range.

Dealer Up Card	Dealer Bust %	Player Advantage % with Basic Strategy
2	35.30%	9.8%
3	37.56%	13.4%
4	40.28%	18.0%
5	42.89%	23.2%
6	42.08%	23.9%
7	25.99%	14.3%
8	23.86%	5.4%
9	23.34%	-4.3%
10, J, Q, K	21.43%	-16.9%
A	11.65%	-16.0%

Figure 3. Player's advantage based on basic strategy, expected value and the dealer's hand. (Blackjack Odds, 2015).

Variance, standard deviation, n-zero and certainty equivalence are specific statistical terms that are important elements to being successful at the game of blackjack. The Blackjack Apprenticeship (2020) website defines variance as “the difference in the expected advantage and the actual results produced.” While dealers lose hands constantly, their overall advantage is realized due to the countless hands that are played each day which reduces the variance in what is expected (casinos making large gains) versus what happens at times (players making large gains). They go on to define standard deviation as “how far or how often an outcome will deviate from the average.” It is very closely related to variance and having a repeatedly large standard deviation could mean you are having the worst day (when the standard deviation is below the average) or the best day (when the standard deviation is above the average) at the casino tables. Casinos may even think a third option is happening- cheating- which will result in an even worse day if discovered. N-zero is a term not known by most players (or even most dealers). It is the number of hands needed before gaining an advantage of one standard deviation. In order to find N-zero and capitalize on the one-standard-deviation advantage, a player must use the same strategy (i.e. hit rules for each scenario), bet in the same manner, and count each hand until the discovery is made. Card counting is greatly discouraged in blackjack (considering the amount of calculations and probability used by casinos for this game, it is very interesting that they are bothered by players strategizing in a similar way) but some players manage to get away with it and n-zero is a part of the information they use to get ahead in the game. Lastly, the website defines certainty equivalence (also known as risk-adjusted return) as “the product of taking the expected win rate and adjusting it based on the level of risk in proportion to the current bankroll and level of risk tolerance." Based on the amount of money a player is willing to use at the table will determine if it makes sense to play. Figure 4 shows examples of how certainty equivalence works. The involvment of these concepts point to how intricate and complicated the game of blackjack actually is.

An easy way to think about the concept behind CE: "I could pay you $100 cash, or you could take a 50/50 chance at $200. Is it worth it? What about a 50/50 chance at $101? 1/10 chance at $20,000? At some points, you will take the risk. At other times, the risk involved isn't justified.

Figure 4. Example of how certainty equivalence works. (Blackjack Apprenticeship, 2020).

Edu-tainment: Game Use in the Classroom

It is clear to see just how important math's role can be in games. In games like blackjack, success is driven by specific math calculations. I believe math education can be just as successful at using games. Some forms of games have been used in classrooms probably for centuries, but it is difficult to find much evidence of its start until computers and games using other technology such as apps were implemented. In Canada, chess has become synonymous with many math classes as a daily activity for learning problem solving and calculating moves among other skills. In parts of Europe, computer programming goes hand in hand with math learning at all grade levels. These are great examples of gaming with a purpose to further math knowledge not just an activity. The distinguishing factors of games are so important that game theory was developed in the early 1900's by Princeton mathematician John Von Neumann. He believed that in order for a game to truly be a game it needs to involve strategy, choices, conflict, and "possibilities for cooperation" (Econlib, 2018).

"Edu-tainment", or educational entertainment, is a term that started being used in the 70's with the introduction of video games. Because our concept and expectations about how people learn are constantly changing and evolving, it is a field that is also very fluid and dynamic and that has adapted well to both technological and educational advancements (Egenfeldt-Nielsen, 2008). Educational learning is a response to a lot of the challenges presented when learning a new subject or skill set. Games have the power to keep its users thoroughly engaged for long periods of time, the problem for designers is to translate this into the educational field.

Egenfeldt-Nielsen's paper on edutainment deals with a lot of the research that has been conducted on the efficacy of playful learning; one of the first studies (conducted in 1981) regarding math-based computer games concludes that the games prove to be "motivating, engaging, and successful in teaching children the planned math concepts." In another study held in 1995, Joseph Betz concluded that the use of games in an engineering technology class proved to be generally more positive when compared to a class not using games. Betz argued that not only were students more engaged in the course, but they also claimed to understand the topic and readings better because of the gaming experience. (Egenfeldt-Nielsen, 2008)

Much more recently, with the refinement of the gaming industry, strategy-based and simulation-type games have expanded the possible subjects and topics being covered by edutainment. Teaching people to code and program through games has proven to be more successful through these more immersive styles of games. The future of educational gaming is focused on the future technologies and what they can offer in terms of experiences. Current games already deal with ethical, identity and responsibility issues, so how much further can the games of the future expand our knowledge and help us become better and smarter humans?

A Lesson in Impactful Classroom Game-Play: Probability 101

Even though mathematical games can remove drudgery from the learning situation, teachers should incorporate mathematical games into their instructional programs and not use games to provide an enjoyable interlude in learning. Teachers should also not use games to provide a break for themselves in teaching. When using mathematical games in class, teachers should join in the games or move around and observe the children at play. They may ask questions to probe children's thought processes and understand their thinking strategies. Students should gain a better understanding of concepts, valuable practice and motivation through these games. They should also help them develop thinking skills and promote problem-solving capabilities. To attain this objective, teachers should select mathematical games that have problems embedded in them so that children can improve their solution strategies by thinking mathematically. For example, in a week-long curriculum I developed to teach the basics of probability, students play the games corners, pig and blackjack to test out the concepts, improve their understanding, and work towards better game strategies with their new found knowledge (see Appendix A-C). They have to make conjectures and try out different strategies in order to complete the daily lessons. The curriculum can be used early on with high school probability and statistics students or towards the end of algebra 1 and geometry courses when year-end math exploration is underway.

Games are also useful for promoting creativity. When children learn mathematics, they are often required to complete exercises set by their teachers or those found in their textbooks. There is no opportunity for them to create their own practice exercises. In the context of games, children can be encouraged to create their own games using the existing game structures. They can do so by using other types of numbers (e.g. decimals instead of fractions) or operations (e.g. addition instead of subtraction) and modifying the rules to simplify the game or to make the game more complex. Children can then exchange their games and play them.

When children play games cooperatively, they have opportunities to listen to others as they share ideas and clarify their thinking. Hence, the teacher can modify games for teams and have mixed-ability groups play games together. Children are usually competitive in nature and there is a strong incentive for players to check one another's mathematics and challenge moves that they think are not valid. The teacher should promote this aspect of playing games as it provides a meaningful context for children to discuss and communicate mathematics. In addition, the teacher should conclude all game sessions with a discussion in which children are encouraged to express their strategies and justify the rules used. Any underlying misconception can then be identified and subjected to peer discussion. The teacher may follow the discussion with a worksheet to consolidate the understanding gained. All of these features are included in my probability game curriculum as well and were invaluable to a successful lesson completion.

In addition, through the dynamic interplay between the cooperative and competitive learning situations, games such as corners from my curriculum can be used to help children develop social skills and even promote physical activity. A child will certainly learn to conform and behave in a civilized way if he or she is to find other children willing to play. Also, basic forms of physical activity like moving your arms or waving hands help with learning, allowing children the opportunity to get all the way up and spark a connection in the brain to help with processing and retention of new information. The physical connection to what the brain needs to know to make the number decision is instant and long lasting, while the laughter and discussions they have well after game time ends also creates a lingering learning effect that lends to all student levels feeling equal and challenged to complete the tasks. Hence, mathematical games do have a valid part to play in mathematics education.

Games for Gaps: Helping Students Play Catch-Up

According to the National Center for Education Statistics, students in the US perform progressively worse in math over time (US News, 2019). Not only is there a decline from elementary to high school math scores but scores overall are no longer on the rise as they were in the 90's (Figure 6). In fact, they have declined at the 8th grade level. I suspect we will see even more of this with the current online learning requirements due to the global spread of the coronavirus.

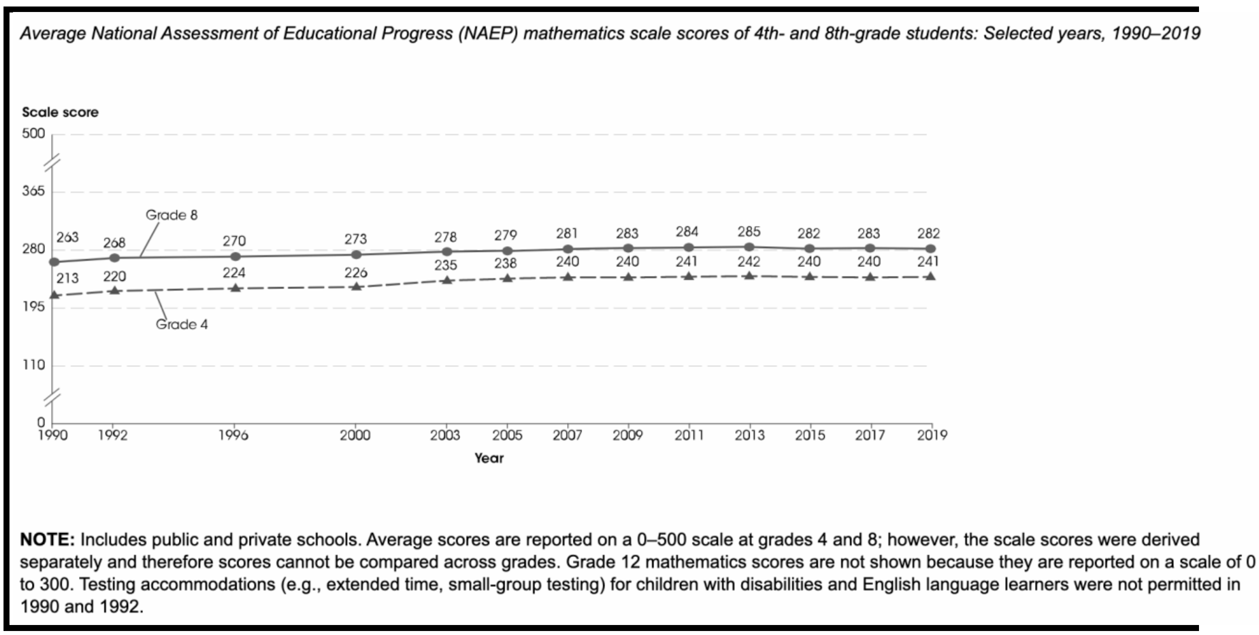


Figure 5. Math Performance for US students
(National Center for Education Statistics)

High school and community college math instructors are forced to handle these declining skills and gaps in learning. Students' behavior is not the best once they get to this point and many have given up on any chance of being successful in a math class.

As the head math teacher at an alternative school in Portland, Oregon, I teach students who are struggling with unlearned or forgotten skills all the time. Many of them are also dealing with the burden of being a student of color, identifying as LGBQT, serious family problems, and/or personal problems that have caused them troubles in school. They are often discouraged about their ability to do well in school and don't see a bright future ahead. I see it as my job to change their thinking and help them realize it is not too late to gain knowledge and be successful in school and in life. Thankfully, my class sizes are small and manageable, so I am able to give them a great deal of one-on-one and small group help. Using games and real-world lessons garner the most attention and retention when I'm working with the classes as a whole. Students are often able to recall the lesson better when they have a memorable experience to go with it.

Many of the teachers I speak with are noticing an increase in the numbers of students who need extra help, as well as a wider gap between the kids who easily understand the math concepts and those who don't. Math games can be an effective strategy for closing the gap, especially games that can be personalized to each individual students' learning path. This way, students can make meaningful gains in knowledge, without always having to be in groups leveled by ability. Within the ability-leveled groups, many kids simply shut down; they have little desire to continue to work hard when they feel they are always behind their peers in class. It would be best to generate a game environment that fosters the constant connections minus the feelings of segregation.

The resounding claim by my students is how much they enjoy the chance to be outside, active and learning all at the same time. It is often forgotten, in the hustle and bustle of meeting all the benchmarks, testing deadlines and chaos of the daily grind in teaching math, that it can still be fun. Kids respond to learning when engaged, challenged and enjoying their tasks. Game-based learning has been an integral part of student development for years, yet we don't give it the support it deserves for the level of knowledge-growth it creates. Educators can start small and work their way up to an entire day. They can begin the process with introductory lesson starter games and work their way into a full math rotation. Eventually a good comfort level will be attained and a full day of learning with math games will be possible.

It takes time to develop the games that work. There needs to be flexibility and freedom to try things, students should be able to provide feedback, and changes should be made where needed. There is no failure in the attempts to generate a game-based learning environment; the only failure comes from never attempting it in the first place. Teachers can work together to tweak ideas and find the most successful tools. The process can get daunting, but it is also fulfilling. In The Role of Games in Mathematics , B. Davies outlines the advantages of using games to teach mathematics as:

- Meaningful situations - for the application of mathematical skills are created by games
- Motivation - children freely choose to participate and enjoy playing
- Positive attitude - Games provide opportunities for building self-concept and developing positive attitudes towards mathematics, through reducing the fear of failure and error
- Increased learning - in comparison to more formal activities, greater learning can occur through games due to the increased interaction between children, opportunities to test intuitive ideas and problem-solving strategies
- Different levels - Games can allow children to operate at different levels of thinking and to learn from each other. In a group of children playing a game, one

child might be encountering a concept for the first time, another may be developing his/her understanding of the concept, a third consolidating previously learned concepts

● Assessment - children's thinking often becomes apparent through the actions and decisions they make during a game, so the teacher has the opportunity to carry out diagnosis and assessment of learning in a non-threatening situation

● Home and school - Games provide 'hands-on' interactive tasks for both school and home

● Independence - Children can work independently of the teacher. The rules of the game and the children's motivation usually keep them on task.

Additional benefits include social interaction for all students and more comfortability for ESL students. The possibilities are endless and the rewards are plentiful.

Conclusion

In many games such as blackjack, the slight chance of winning and the fun of playing is enough to keep players engaged and coming back for more. Even though people know casino games and the lottery are structured for them to lose, it does not stop millions from being transacted in those arenas. It is my hope that games offer students a major chance of success in their math journey and keep them engaged and ready for more. If teachers are willing to think outside of the box and be more innovative with their lesson plans, we could see some real success in students that would otherwise continue to fail. Games will not work for every student just like there is no other method that works for everyone, but it will definitely make a difference for some and add to the number of students who are achieving their mathematical goals.

Part Two:
Curriculum

Teaching Probability 101 Through Games

Overview of the Curriculum

As a math teacher, I have always looked for ways to garner excitement in my students for a subject many of them find difficult and frustrating. It is a hard task and became even harder when I started teaching at an alternative school where some students had moved around so much that they were not learning much during the transitions or had circumstances that preventing them from being in school for long stretches of time. One tactic that always puts students at ease and on level ground is game days. So a couple of years ago, I explored ways to make this more of a tool during the year. After using several lessons i found from others, I decided to develop my own. Based on the subjects I teach, probability was a great fit for exploring learning through games and my probability 101 curriculum was born!

Grade Level:

Adaptable for all high school grades. Written for algebra 1 or geometry course exploring new math concepts at the end of the school year.

Time Needed:

- Approximately five hours, split between five days.
- Each part of the task has suggested time allotments. Adjust to meet the needs of your class.

Materials Needed:

- Worksheets for all students
- Game rules
- Journals/Paper (for recording strategies and game results)
- Dice (enough for each pair of students)
- Playing cards (enough for each pair of students)

Goals:

Content Goals

- Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events. (CCSS CP.A.1)
- Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. (CCSS CP.A.2)
- Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the sae as the probability of A, and the conditional probability of B given A is the same as the probability of B.
- Connect the concepts and processes of outcomes and basic probability.

Math Practices and Critical Literacy Goals

- Make sense of problems and persevere in solving them. (CCSS HS.MP.1)
- Construct viable arguments and critique the reasoning of others. (CCSS HS.MP.3)
- Practice making decisions about data

Required Skills/Content:

- Students will need some exposure/working knowledge of probability (given the week prior in my project).
- Students will need to know types and amounts of suits in card deck. This information should be included in the worksheet packets or posted in the classroom.

General Teacher Notes and Format:

- This lesson includes games. Make sure you know how to play them first then be able to demonstrate how to play them with another student in front of the class. This will go a long way to helping all students understand the rules. Allow time for students to read the rules as well.
- You do not need to control the ideas that students generate. Your role will be to observe and facilitate critical thinking in regards to strategies and answers to the worksheet questions. See if students can name the probability concepts at work in the games (independent versus dependent events, sample spaces, conditional probability).
- This lesson plan can be used for in-person or remote instruction. If implementing remotely, share any slides and GoogleDocs with students so they can follow along (particularly the game rules).

Schedule of Activities (Recommended):

Day 1:	**Introduce the Context**
	Basic Probability
Day 2:	**Activity: The Game of Corners**
	Independent/Dependent Events
	Lesson Extension: Monty Hall problem
Day 3:	**Activity: The Game of Pig**
	Sample Spaces
	Lesson Extension: Playing with 2 dice
Day 4:	**Activity: The Game of Blackjack**
	Conditional Probability
	Lesson Extension: Allowing more than two players
Day 5:	**Lesson Conclusion**

Day 1: Introduce the Context

Objective:

To explore basic probability

Lesson Flow:

- review the definition and uses for probability
- give real world examples of probability
- watch introductory video
- if time permits, start teaching students how to play the games

Day 2: The Game of Corners

Objective:

To explore independent/dependent events

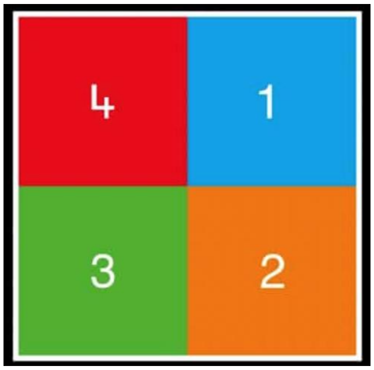
Lesson Flow:

- demonstrate how to play the game with the entire class (play outside if possible)
- play without replacement (entire class)
- play with replacement (entire class)
- complete corners worksheet

Game Rules - Corners:

1. Choose a player to go in the middle. He/she will close their eyes and count slowly and loudly backward from ten to zero.
2. While they are counting, everyone must find a corner and stay there or they are out.
3. The player in the middle points to the corner of his/her choice, and then opens their eyes.
4. Anyone standing in the designated corner must sit down.
5. When only one player is left standing, he/she becomes the counter for the next round.
6. When the game gets down to four people or less, each must choose a different corner.

Worksheet: The Game of Corners



1. What is the probability of losing in the first round of corners?
2. What is the probability of staying in the game in the first round of corners?
3. What is the probability of losing in the second round of corners if the same corner can't be called again?
4. What is the probability of losing in the second round of corners if the same corner can be called again?
5. What is the probability of losing in the third round of corners if the same corner can't be called again?
6. What is the probability of losing in the third round of corners if the same corner can be called again?
7. What do you notice about the chances of losing/winning when the same corner can't be called again in comparison to when the corners can be repeated?

Answer Key: The Game of Corners

1. What is the probability of losing in the first round of corners? 1/4
2. What is the probability of staying in the game in the first round of corners? 3/4
3. What is the probability of losing in the second round of corners if the same corner can't be called again? 1/3
4. What is the probability of losing in the second round of corners if the same corner can be called again? 1/4
5. What is the probability of losing in the third round of corners if the same corner can't be called again? 1/2
6. What is the probability of losing in the third round of corners if the same corner can be called again? 1/4
7. What do you notice about the chances of losing/winning when the same corner can't be called again in comparison to when the corners can be repeated?

When the same corner can't be called again, the probability of losing increases. Also, the game ends faster.

Lesson Extension:

- Introduce the Monty Hall problem and play a Monty-Hall-type version of corners

  Introductory Video: The Monty Hall Problem

- During the video, pause it before the answer/explanation is given about the probability of selecting the correct door. Allow students to reason about the answer and give their thoughts on it.
- After some discussion about the Monty Hall problem and its relationship to dependent events, play a "triangular" form of corners where all the students choose between two of three corners (one corner will not be used for this version of the game). Those corners will be the "goats". The empty corner will be the prize. Allow students to take turns being the middle person and attempt to win by selecting where their classmates are not at, keeping in mind how the Monty Hall problem works.

Day 3: The Game of Pig

Objective:

To explore sample spaces

Lesson Flow:

- demonstrate how to play the game with another student (take several turns and land on one at least once so students see how the game goes)
- have students decide on a number of rolls to take each time before they stop accumulating points
- have students pair up and play two rounds (have loser from first round change their strategy for the second round; winner can also change their strategy if they think there is a better way to win)
- complete pig worksheet

Game Rules - Pig:

1. Choose a player to go first. That player throws a die and scores as many points as the total shown on the die provided the die doesn't show a 1. The player may continue rolling and accumulating points (but risk rolling a 1) or end his/her turn
2. If the player rolls a 1 his/her turn is over. He/she loses all points accumulated for that turn.
3. The next player takes their turn and does the same.
4. Play passes from player to player until a winner is determined.

How do you win?

The first player to accumulate 100 or more points wins the game.

Variation: Two-Dice Pig

This variant is the same as Pig, except two standard dice are rolled. If neither shows a 1, their sum is added to the turn total. If a single 1 is rolled, the player scores nothing and the turn ends. If two 1s are rolled, the player's entire score is lost and their turn ends.

Variation: Big Pig

This variant is the same as Two-Dice Pig, except rolling double 1s ends the player's turn, scores 25 points, and eliminates any other points the player may have accumulated that turn. If any other doubles are rolled, the player adds twice the value of the dice to the turn total.

Worksheet: The Game of Pig

1. What is the probability of rolling a 1 in pig?
2. What is the probability of rolling a 2 in pig?
3. What do you notice about the probability of rolling each number?
4. What would happen to this probability if we added a second dye to roll with the first?
5. What was the average number of times it took for a 1 to be rolled?
6. What was your strategy to play pig?
7. Did you win the first round? Why do you think that occurred?
8. Did you win the second round? What did you do differently?

Answer Key: The Game of Pig

1. What is the probability of rolling a 1 in pig? 1/6

2. What is the probability of rolling a 2 in pig? 1/6

3. What do you notice about the probability of rolling each number?

   The probability is the same for each number.

4. What would happen to this probability if we added a second dye to roll with the first?

It would not be possible to get the number 1 anymore. The probability of getting 2 would go from ⅙ to 1/36 (decreases). The probabilities of the other numbers (3-6) decreases as well, and now other numbers get introduced (7-12).

5. What was the average number of times it took for a 1 to be rolled? Answers will vary.

6. What was your strategy to play pig? Answers will vary.

7. Did you win the first round? Why do you think that occurred? Answers will vary.

8. Did you win the second round? What did you do differently? Answers will vary.

Lesson Extension:

- Play a third time with 2 dice. Explore the differences in the new sample space and the new probabilities the additional die creates.
- Explain the rules of Two-Dice Pig from the game instructions page.
- Have students come up with the new sample space (write it up on the board as they give the answer).
- Ask students how the new sample space changes their chances of winning/losing. Use the worksheet to ask questions with the new probabilities (questions 2, 3, and 5).

Day 4: The Game of Blackjack

Objective:

To explore conditional probability

Lesson Flow:

- demonstrate how to play the game with another student (play several hands while explaining the procedure of taking additional cards and the differences between being the dealer and player)
- have students decide what card total they will stop at each hand
- have students pair up and play 20 hands or until you stop play to complete the worksheet
- about midway through, allow students to change their strategy if desired

Game Rules - Blackjack:

https://www.blackjackinfo.com/blackjack-rules/

<u>Worksheet: The Game of Blackjack</u>

1. What is the objective of this game (how do you win)?

2. What is the probability of drawing a 10 for your second card when you have a 10 for the first card?

3. What is the probability when you don’t have a 10 for the first card?

4. What is the probability of getting 21 with 2 cards when you have 10 as your first card?

5. Do you notice any advantages to playing second? Explain.

6. What is the probability of busting with an additional card if you have 16?

7. Can you think of any reason why hitting 16 is a good idea but hitting 17 is not?

8. What was your strategy to play 21?

9. What effect would using multiple decks in the game have?

Answer Key: The Game of Blackjack

1. What is the objective of this game (how do you win)? To get 21 or as close to it as possible without going over; to get a higher number than the dealer without going over 21

2. What is the probability of drawing a 10 (any card with the value of 10) for your second card when you have a 10 for the first card? 52-1= 51 total possibilities, 15 ways to get 10 (10's, J's, Q's, and K's- 4 each except the 10 you already have), so 15/51
*Teacher Note: Go over this question with the class as whole. Remind students not to include the first card here or on the next few questions since it's no longer a possibility.

3. What is the probability when you don't have a 10 for the first card? 16/51

4. What is the probability of getting 21 with 2 cards when you have 10 as your first card? 52-1= 51 total possibilities, need an ace to get to 21 so 4/51

5. Do you notice any advantages to playing second? Explain. Yes. The first player can bust and lose without the second player (dealer) even going. Teacher Note: You can mention this is one of the reasons casinos make so much money. Many players bust and have their money taken away before the dealer even plays their hand.

6. What is the probability of busting with an additional card if you have 16 (with 2 cards- 10 and 6)?

52-2= 50 total possibilities; 6, 7, 8, 9, and 10 will cause the person to bust and there are 4x4=16 (6-9) + 4x4 (10 and face cards) = 32 - 2 (the 6 and 10 you already have) = 30/50

7. Can you think of any reason why hitting 16 is a good idea but hitting 17 is not?

Answers will vary. More chances to bust on 17 than 16.

8. What was your strategy to play 21? Answers will vary.

9. What effect would using multiple decks in the game have? Keeps people from counting cards. Increases the chances of getting a 10 since there are so many ways to get them.

Lesson Extension:

- Allow more than two students to play together while taking note of the dealer's increased advantage with each additional player.
- Use the same rules from the earlier rounds. The dealer will let each player complete their hand, moving from left to right. He or she will continue to go last.
- Discuss how this works in casinos and how it affects their bottom line (more money!)

Day 5: Lesson Conclusion

Objective:

To explore implications and tie together concepts

Lesson Flow:

- complete any unfinished worksheets
- discuss larger implications of probability (casinos, insurance, lottery, etc.)
- play additional rounds of pig and blackjack if time permits

Think-Pair-Share Ideas: Have partnered students discuss their strategy reasonings then settle on one to share with the rest of the group. Allow students to debate why their choice is best. In the case of pig, you can share that according to how probability works you should stop at the 5th roll (since, theoretically, every sixth roll should produce a 1). In the case of blackjack, you can share that stopping at 17 is the strategy used at casinos.

Curriculum Reflection

It was fortunate that I was able to pilot my activities once before trying them out on my class of algebra students (the intended audience). Since I also happen to teach a statistics and probability course, I decided to run the curriculum on those students first. It was a success and allowed me to make adjustments before completing the actual run with a couple of Algebra 1 classes that my colleague teaches. Two classes of Algebra 1 completed the week-long curriculum. I took over one of the classes for the week and the regular teacher taught the other. My class was characterized by the teacher as hard-woking but quiet. She characterized the class she taught as talkative but struggles to complete assignments. Each class consisted of about 12 students and all were 9th graders. After my class got over the initial shock that they weren't being taught by their regular teacher for the week, the curriculum was underway.

Even though the classes were different, we observed some of the same processes in both. Students were ready for corners, challenged by pig, and frustrated by the probability concepts in blackjack. After answering the first few worksheet questions, most students were able to differentiate and define the independent versus dependent events in the different versions of corners. This game and the corresponding worksheet were great for getting the curriculum started and boosting the students' confidence from the start. Students came to class excited to see what was next the following day when we played pig. They were very methodical in choosing their strategy for this game and quite talkative during the rounds. They found the worksheet questions more challenging but asked good questions to help them to get to the answers. With blackjack there was some frustration around the concept of conditional probability. Some students had difficulty grasping the changes in probability as a result of taking away cards. For instance, when asked the probability of getting 21 with 2 cards when you have 10 as the first card, students struggled with combining the probabilities of both cards. In the future, I will save these type of questions until after we go through conditional probability a bit more.

At the end of the week, I interviewed the Algebra 1 teacher and she gave me some insight on the impact of the games for her classes. She noted that her more successful students were more talkative than normal and her under-performing students were more engaged. She also had a higher submission rate of completed worksheets. The games were a great follow-up to the previous week's introduction to basic probability and helped solidify the vocabulary and ideas. As she dives deeper into the concept of probability with the class over the following weeks, she is looking forward to being able to use the games as a reference for helping students remember how the concepts work. The one suggestion she gave was to give rewards (e.g. bonus points, homework pass) or prizes (e.g. candy) to the winners of the games to increase interest even more.

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